Exercise 61

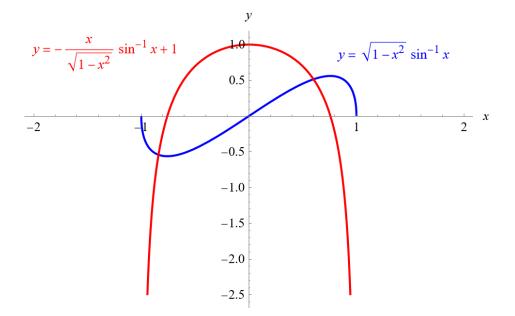
Find f'(x). Check that your answer is reasonable by comparing the graphs of f and f'.

$$f(x) = \sqrt{1 - x^2} \arcsin x$$

Solution

Use the product rule, the chain rule, and the derivatives of the inverse trigonometric functions listed on page 214.

$$\begin{aligned} \frac{df}{dx} &= \frac{d}{dx} \left(\sqrt{1 - x^2} \arcsin x \right) \\ &= \left(\frac{d}{dx} \sqrt{1 - x^2} \right) \arcsin x + \sqrt{1 - x^2} \left(\frac{d}{dx} \arcsin x \right) \\ &= \left[\frac{1}{2} (1 - x^2)^{-1/2} \cdot \frac{d}{dx} (1 - x^2) \right] \arcsin x + \sqrt{1 - x^2} \left(\frac{1}{\sqrt{1 - x^2}} \right) \\ &= \left[\frac{1}{2} (1 - x^2)^{-1/2} \cdot (-2x) \right] \arcsin x + 1 \\ &= -\frac{x}{\sqrt{1 - x^2}} \arcsin x + 1 \end{aligned}$$



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