Exercise 61

Find $f'(x)$. Check that your answer is reasonable by comparing the graphs of f and f'.

$$f(x) = \sqrt{1-x^2}\arcsin x$$

Solution

Use the product rule, the chain rule, and the derivatives of the inverse trigonometric functions listed on page 214.

$$\begin{aligned}
\frac{df}{dx} &= \frac{d}{dx}\left(\sqrt{1-x^2}\arcsin x\right) \\
&= \left(\frac{d}{dx}\sqrt{1-x^2}\right)\arcsin x + \sqrt{1-x^2}\left(\frac{d}{dx}\arcsin x\right) \\
&= \left[\frac{1}{2}(1-x^2)^{-1/2}\cdot\frac{d}{dx}(1-x^2)\right]\arcsin x + \sqrt{1-x^2}\left(\frac{1}{\sqrt{1-x^2}}\right) \\
&= \left[\frac{1}{2}(1-x^2)^{-1/2}\cdot(-2x)\right]\arcsin x + 1 \\
&= -\frac{x}{\sqrt{1-x^2}}\arcsin x + 1
\end{aligned}$$